LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2023**

PST 2502 – TESTING STATISTICAL HYPOTHESES

Date: 02-05-2023 Dept. No. Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

SECTION-A

Answer ALL the questions. **(10 x 2 = 20)**

1. Distinguish between simple and composite hypotheses.
2. State Generalized Neyman-Pearson Theorem.
3. Explain briefly an unbiased test and describe its applications
4. Define Multi parameter Exponential Family.
5. When do we say that a test ϕ has Neyman Structure?
6. What are nuisance parameters and how do you remove them?
7. What is maximal invariant function?
8. Briefly explain the principles of LRT.
9. Give an example of a group of distributions with location changes.
10. Define p-value and provide any one use of p-value.

SECTION-B

Answer any FIVE questions. **(5 x 8 = 40)**

11. Let $X_1, X_2, \ldots, X_n$ be iid B(1 ,p) random variables. Find the Most powerful test function of level α for testing $H_0: p = p_0$ Vs $H_1: p = p_1$ $(p_0 > p_1)$.
12. Give an example for Non exponential family of distribution possessing MLR property and prove.
13. Why do we require bounded completeness to prove similar tests to have Neyman structure? Explain.
14. Consider the one parameter exponential family of distributions. Obtain the UMPT of level α for testing the one-sided testing hypothesis.
15. Let β denote the power of a most powerful test of level α for testing simple hypothesis H_0 against simple alternative H_1. Prove that (i) $\beta \geq \alpha$ and (ii) $\alpha < \beta$ unless $p_0 = p_1$.
16. Using a random sample from U(0, θ) derive UMPT for H: $\theta \geq \theta_0$ versus K: $\theta < \theta_0$.
17. Obtain the Likelihood Ratio Test for equality of means of 'k' normal populations with a common variance.
18. Derive the Locally Most Powerful test for testing $H_0 : p = 1$ Vs $H_1 : p < 1$ based on a random sample of size n from $f(x,\theta) = pf_1(x,\theta) + (1-p)f_2(x,\theta)$, where f_1 and f_2 are known pdf's.

SECTION-C

Answer any TWO questions. **(2 x 20 =40)**

19. State and prove the existence, necessary and sufficiency parts of Neyman-Pearson Fundamental Lemma.
20. (a)Derive a UMP test of level α for testing $H_0 : \theta \leq \theta_0$ Vs $H_1 : \theta > \theta_0$ for the family of densities $\{f(x,\theta), \theta \in \Theta\}$ that possess MLR in T(x). Show that the power function of the above testing problem increases in θ

    b.) Show that any UMP test is always UMPUT. (16+4)

21. Consider a one parameter exponential family with density $f(x) = c(\theta)e^{Q(\theta)T(x)}h(x)$. Assume $Q(\theta)$ is strictly increasing in θ. Show that for testing $H_0 : \theta \le \theta_1 \quad or \quad \theta \ge \theta_2$ Vs $H_1 : \theta_1 < \theta < \theta_2$, prove that there always exist UMP test of level α and is of the form $\phi^*(x) = \begin{cases} 1 & if \quad c_1 < T(x) < c_2 \\ \gamma_i & if \quad T(x) = c_i, \quad i = 1,2 \\ 0 & otherwise \end{cases}$

where the constants are selected so that $\beta_{\phi^*}(\theta_1) = \beta_{\phi^*}(\theta_2) = \alpha$.

22. (a) Let X and Y be independent Binomial variables with parameters (m, p_1) and (n, p_2) respectively, where m and n are assumed to be known. Derive a conditional UMPUT of size α for testing $H_0 : p_1 \le p_2$ Vs $H_1 : p_1 > p_2$.

(b) Let $X_1, X_2, \ldots X_n$ be iid $N(\mu, \sigma^2)$, Find the shortest length confidence interval for μ with level $1-\alpha$ based on a minimal sufficient statistic. (10+10)

$$$$$$$